

NAME: _____

Test Prep 2

There are two quick solving problems on this page (all the integrals should be quick; if not, then you made a mistake). If you finish this page, try the extra problems on the back (for an extra point). You have 10 minutes!

1. (Separable) Find the general explicit solution to $\frac{dy}{dt} = \frac{t}{y^2}$

Sol'n:

$$\begin{aligned}y^2 dy &= t dt \\ \frac{1}{3}y^3 &= \frac{1}{2}t^2 + C_1 \\ y^3 &= \frac{3}{2}t^2 + C_2 & C_2 = 3C_1 \\ y &= \left(\frac{3}{2}t^2 + C_2\right)^{1/3}\end{aligned}$$

Thus,

$$y = \left(\frac{3}{2}t^2 + C\right)^{1/3}$$

2. (Integrating Factor) Find the explicit general solution to $\frac{dy}{dt} + \frac{1}{t}y = t$ (You may assume $t > 0$).

Sol'n:

The integrating factor is $\mu(t) = e^{\ln(t)} = t$. Multiplying gives

$$\begin{aligned}t \frac{dy}{dt} + y &= t^2 \\ \frac{d}{dt}(ty) &= t^2 \\ ty &= \frac{1}{3}t^3 + C_1 \\ y &= \frac{1}{3}t^2 + \frac{C_1}{t}\end{aligned}$$

Thus,

$$y = \frac{1}{3}t^2 + \frac{C}{t}$$